

Numerical simulation of laminar vortex shedding in the wake of circular cylinders

Lucy Guo
Lguo13@lsu.edu

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Keith Gonthier

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Introduction

When a bluff body such as a circular cylinder interacts with a surrounding flow, depending on the geometry of the body and the Reynolds number, instabilities within the flow may arise [1]. An adverse pressure gradient created by the bluff body can cause the boundary layer to separate from the cylinder, generating an oscillating wake behind the body known as vortex shedding [2]. If the frequency of the vortex shedding aligns with the natural frequency of the body, the body will begin to violently oscillate. In the real world, the occurrence of this phenomenon has even resulted in structural failures [3]

In the following report, a simple 2D simulation of a 1m diameter cylinder in a laminar flow was modeled. The Reynolds number of the flow was varied from 50 to 150, and corresponding flow characteristics such as lift coefficient, drag coefficient and Strouhal number were calculated and analyzed. By understanding the relationship between Reynolds number and the resulting vortex shedding characteristics, informed design decision can be made for objects of similar geometry in the real world, allowing engineers to prevent structural failure due to vortex shedding for these structures.

Methods

For this simulation, a 1m diameter cylinder was modeled on a 50m x 20m 2D plane based on the dimensions and layout shown

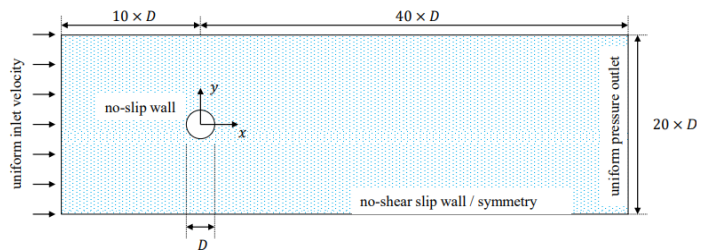


Figure 1: Cylinder Model Dimensions [3]

in figure 1 to the right. The flow was assumed to be a 2D, incompressible, isothermal, laminar flow exhibiting transient behavior with viscous effects playing a significant role. The software used to analyze flow characteristics was ANSYS 19.2.

To generate the mesh, the MultiZone Quad/Tri Method was used with an element size of 0.15m. The incoming fluid velocity at the inlet was set 5m away at 1 m/s and the nondimensional fluid used had a density of 1 kg/m³ and a viscosity of 0.01 kg/m-s to represent a Reynolds number = 100. This viscosity was later modified to simulate Re= 50 and Re=150. For the simulation, the PISO algorithm was used to ensure that the Pressure and Momentum Discretizations had Second-Order formulations and a convergence criterion of 0.0001 was used for the residuals.

Results

Figure 2 depicts the flow characteristics: Wake Cross Velocity, Drag Coefficient, and Lift Coefficient around the cylinder for Reynolds number = 100 as a function of dimensionless flow time. After approximately 50 Flow Time all three characteristics reach a steady oscillatory state. The average Lift Coefficient oscillates around zero while the Drag Coefficient oscillates around 1.41. To estimate the Strouhal number, the Fast Fourier Transform (FFT) was utilized to transform the time data of the wake cross velocity to frequency data. (Figure 3) A Strouhal number of 0.166 was found for Re=100.

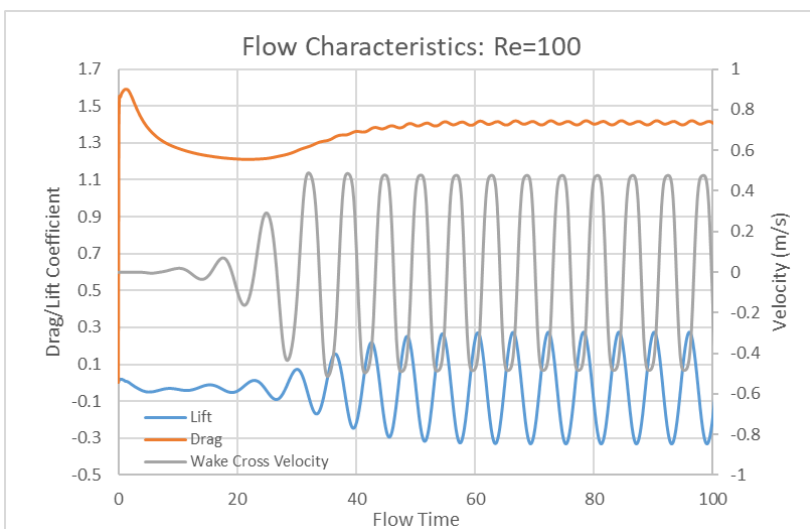


Figure 2: Flow Characteristics at Re=100

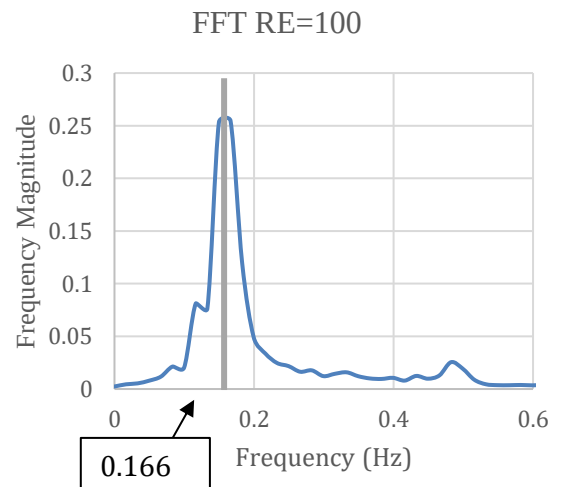


Figure 3: Strouhal Number Re=100

The simulation process was repeated for $Re=50$ and $Re=150$, and the Strouhal number, Drag Coefficient Average, Drag Coefficient Amplitude and Lift Coefficient Amplitude were listed in Table 1 below.

RE	St number	Cd avg	Cd amp	CL amp
50	0.1248	1.48567	0.00007227	0.00415
100	0.166	1.41	0.011677	0.3028
150	0.187	1.34	0.018626	0.38673

Table 1: Flow Characteristics at $Re=50,100,150$

The Strouhal number as a function of Reynolds number was depicted in Figure 4 below and plotted in comparison with the following equation for the Strouhal number: $St \approx (0.195 * (1 - 20.1/Re))$, based on previous literature [3].

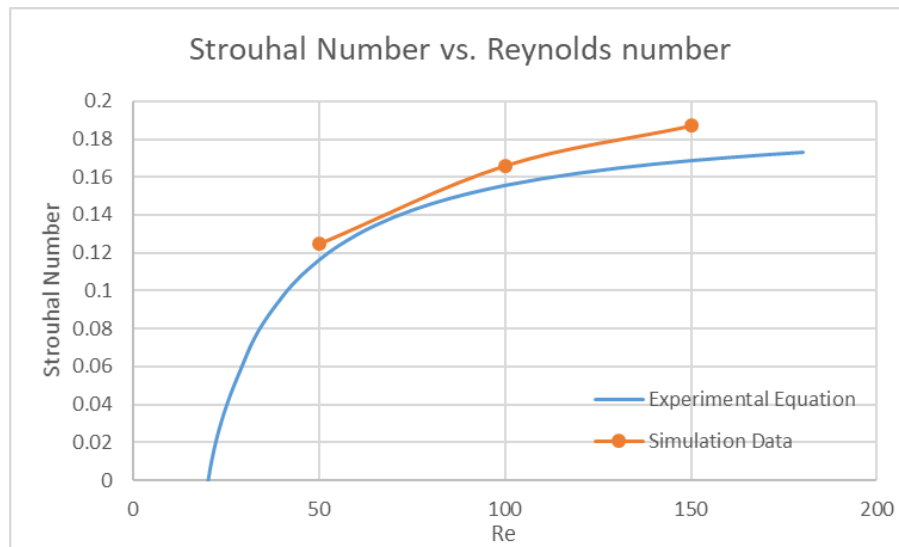


Figure 4: Strouhal Number vs. Reynolds Number: Simulation and Experimental Data

In Figure 5, the average drag coefficient was plotted in relation to Reynolds number and the following linear equations was found: $Cd_{avg} = -0.0015(Re) + 1.5576$

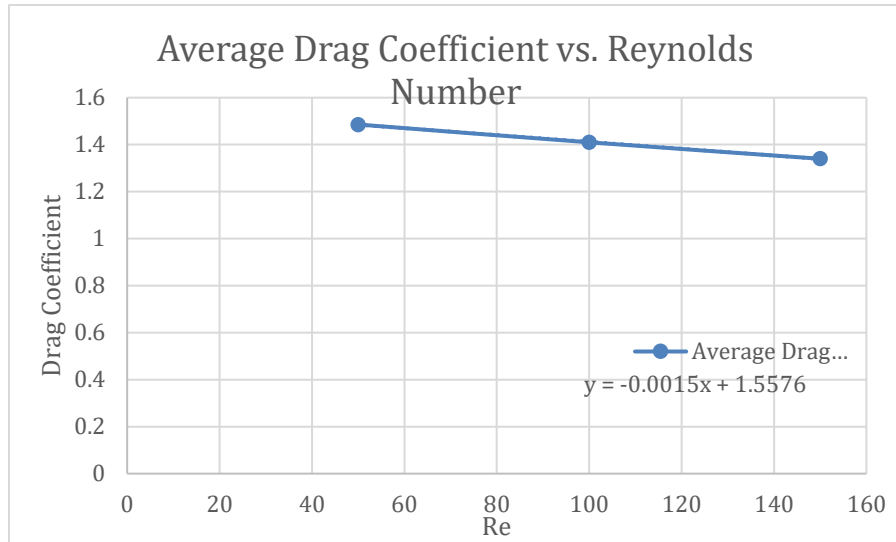


Figure 5: Average Drag Coefficient vs Re: Data and Trendline

Plot correlations for Drag Coefficient Amplitude and Lift Coefficient Amplitude as a function of Reynolds number were also graphed (Figure 6), and 3rd order polynomial trendline equations were found for both data sets. The equations for Lift and Drag Coefficient Amplitudes are as follow:

$$\text{Lift Amplitude} = -4\text{E-}05(\text{Re})^2 + 0.0124(\text{Re}) - 0.5092$$

$$\text{Drag Amplitude} = -9\text{E-}07(\text{Re})^2 + 0.0004(\text{Re}) - 0.0162$$

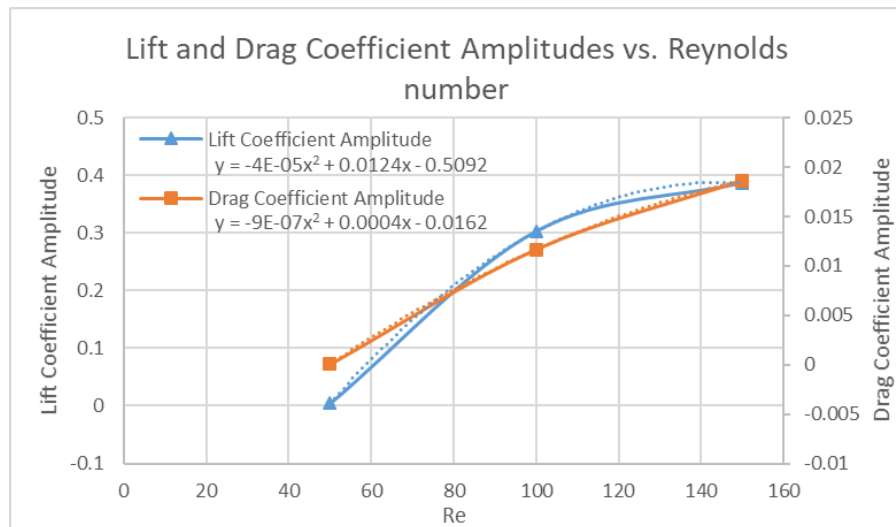


Figure 6: Lift and Drag Coefficient Amplitudes vs Re: Data and Trendlines

Using the Trendline Equations for Lift and Drag Coefficient Amplitudes, a minimum $Re=45$ was calculated to be the critical Reynolds number in which no oscillation occurred (Appendix A).

Discussion

Looking at the results of the experiment it is clear that as Reynolds number increases, the Strouhal number will increase as well. This makes intuitive sense given that St Number is directly related to shedding frequency, which would increase as the Reynolds number of the flow increased. Comparing the Strouhal number found in this simulation to previous literature, the data points closely mirror the trendline of the experimental equation: $St \approx (0.195 * (1 - 20.1/Re))$. Examining other experiments conducted for $Re=100$, St Number and Drag Coefficient Average were found to be 0.1569 and 1.3353 respectively [4]. These numbers are also close to the simulated numbers of 0.166 and 1.41 found in this experiment.

For Average Drag Coefficient, a decreasing linear relationship was observed as the Reynolds number increased following the equation: $C_{davg} = -0.0015(Re) + 1.5576$. This relationship also makes intuitive sense, given that Average Drag coefficient is related to drag force which, at low Reynolds number for blunt bodies, is highly related to Pressure Drag. As the Reynolds number is increased, it causes pressure drag to decrease due to the boundary layer separating closer to the rear area of the cylinder. This in turn results in a decreasing drag coefficient as shown in the simulation. According to experiments in laminar flow for a cylinder, the Drag Coefficient should be around 1.2, which is close to the 1.34-1.48 range found in this simulation [5].

Looking at other literature, the critical Reynolds number resulting in two-dimensional oscillatory flow is found to be around $Re=47$, with various other authors identifying the number to fall around numbers including $Re= 44$ and $Re= 46.4$. [6] These findings agree with the results that were calculated in Appendix A using the similar method of extrapolating Lift and Drag Coefficients until they reached zero. Furthermore, according to Kianifar, for flow in the range of $5 \leq Re \leq 45$, the wake is characterized by a symmetric pair of vortices. When the Re is further increased, the wake becomes unstable. [1] This unstable behavior is displayed in the oscillatory characteristic that appears from the drag and lift coefficient amplitudes of this simulation that are conducted at Reynolds numbers above the critical number.

Conclusions

The following report sought to determine the relationship between Reynolds number and the resulting flow characteristics of a cylinder under laminar flow. Based on the simulations run, a negative linear relationship between average drag coefficient and Reynolds number was calculated as: $C_{davg} = -0.0015(Re) + 1.5576$, and a critical Reynolds number of $Re=45$ was determined for oscillatory behavior to be exhibited. Based on these results, one can determine the drag forces experienced by geometrically similar structures based on the Reynolds numbers expected for these structures to experience. Using this information, additional designs can be incorporated into these structures to both withstand and lessen the influence of drag.

References

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Appendix A

Drag Amplitude vs. Re: Based on Trendline Equation

Re	Drag Amp
45	-2.25E-05
45.1	9.391E-06
45.2	4.1264E-05
45.3	7.3119E-05
45.4	0.00010496
45.5	0.00013678
45.6	0.00016858
45.7	0.00020036
45.8	0.00023212

$$\text{Drag Amplitude} = -9\text{E-}07(\text{Re})^2 + 0.0004(\text{Re}) - 0.0162$$

Lift Amplitude vs. Re: Based on Trendline Equation

Re	Lift Amp
48	-0.00616
48.1	-0.0053
48.2	-0.00445
48.3	-0.0036
48.4	-0.00274
48.5	-0.00189
48.6	-0.00104
48.7	-0.00019
48.8	0.000662
48.9	0.001512
49	0.00236

$$\text{Lift Amplitude} = -4\text{E-}05(\text{Re})^2 + 0.0124(\text{Re}) - 0.5092$$